Control of Turbulent Transport: Less Friction and More Heat Transfer

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Abstract

Because of the importance of fundamental knowledge on turbulent heat transfer for further decreasing entropy production and improving efficiency in various thermo-fluid systems, we revisit a classical issue whether enhancing heat transfer is possible with skin friction reduced or at least not increased as much as heat transfer. The answer that numerous previous studies suggest is quite pessimistic because the analogy concept of momentum and heat transport holds well in a wide range of flows. Nevertheless, the recent progress in analyzing turbulence mechanics and designing turbulence control offers a chance to develop a scheme for dissimilar momentum and heat transport. By reexamining the governing equations and boundary conditions for convective heat transfer, the basic strategies for achieving dissimilar control in turbulent flow are generally classified into two groups, i.e., one for the averaged quantities and the other for the fluctuating turbulent components. As a result, two different approaches are discussed presently. First, under three typical heating conditions, the contribution of turbulent transport to wall friction and heat transfer is mathematically formulated, and it is shown that the difference in how the local turbulent transport of momentum and that of heat contribute to the friction and heat transfer coefficients is a key to answer whether the dissimilar control is feasible. Such control is likely to be achieved when the weight distributions for the stress and flux in the derived relationships are different. Secondly, we introduce a more general methodology, i.e., the optimal control theory. The Fréchet differentials obtained clearly show that the responses of velocity and scalar fields to a given control input are quite different due to the fact that the velocity is a divergence-free vector while the temperature is a conservative scalar. By exploiting this inherent difference, the dissimilar control can be achieved even in flows where the averaged momentum and heat transport equations have the same form.

Keywords: turbulent convective heat transfer, skin friction, control, analogy, turbulent Prandtl number
1. Introduction

Among various technological issues toward a future sustainable society, those related to the conversion, transportation, storage and utilization of energy are crucial. The general strategies seem to have been established; e.g., accelerating the introduction of renewable energy, making the energy utilization efficiency even more higher, and developing safe carbon capture and storage technologies. When shaping these future goals, however, it should be noted that interface phenomena actually play a central role in deciding the efficiency and carbon intensity of devices, equipment, plants, and processes. They are, for example, surface friction, heat and mass transfer, and catalytic, electrochemical, photosynthetic and photovoltaic processes, to name a few. Although the ideal efficiency and performance of an energy system can be described by thermodynamic analyses, there always exist irreversible losses (i.e., entropy production) in transport processes due to nonequilibrium temperature, concentration, and electrochemical potentials. To enhance or suppress these fundamental processes and reduce thermodynamic losses would potentially lead to enormous benefit for the human society through substantial energy saving and carbon emission reduction. In particular, enhancement of heat and mass transfer processes in numerous thermo-fluid systems such as heat exchangers, power plants and production processes should give tremendous impact on not only energy utilization, but also economy and the environment.

In this paper, we revisit a fundamental question whether heat transfer can be enhanced while reducing momentum transfer. The latter condition is to keep the work required for driving a fluid flow moderate or even reduced, since it is irreversibly dissipated into heat. We focus upon turbulent convective heat transfer rather than laminar one because of its wide range of applications and also of inherent difficulty in controlling turbulence.

Although general argument of heat transfer enhancement involves a variety of geometrical configurations (see, e.g., Bergles [1]), we consider one of the most canonical thermo-fluid systems, namely, fluid flow with heat transfer in a straight and smooth channel. The fluid properties are assumed constant and the temperature is a passive scalar throughout this paper, so that any buoyancy effect does not arise. Such a simple flow system offers a chance for us to investigate essential aspects of turbulent heat transfer mechanism. In order to drive a flow, a pumping power $W_p^*$ is required to oppose the wall skin friction $\tau_w^*$, where a variable with an asterisk represents a dimensional quantity. Suppose the flow is in a fully developed
state, the applied pumping power is eventually transformed to heat via the viscous dissipation \( \varepsilon^* \). This means that this process is unavoidably associated with the entropy generation of \( \Delta s_f^* = \varepsilon^*/T_f^* = W_p^*/T_f^* \), where \( T_f^* \) is the characteristic fluid temperature.

Similarly, assuming the heat flux \( q^* \) from a heated wall to a fluid, the entropy generation due to the heat transport can be estimated as \( \Delta s_h^* = q^*(T_f^* - T_w^*) \), where \( T_w^* \) is the characteristic wall temperature, which is supposed to be higher than the fluid temperature. Therefore, the total entropy production rate \( \Delta s^* \) is estimated as:

\[
\Delta s^* = \Delta s_f^* + \Delta s_h^* = \frac{W_p^*}{T_f^*} + q^* \left( \frac{1}{T_f^*} - \frac{1}{T_w^*} \right) = \frac{r_w^* U_b^*}{T_f^*} + h^* \Delta T^*,
\]

where \( U_b^* \) is the bulk mean velocity, while \( h^* = q^*/\Delta T^* \) is the heat transfer coefficient based on the temperature difference between the wall and the fluid, \( \Delta T^* = T_w^* - T_f^* \). From Eq. (1), there exist only two possible strategies to suppress the entropy generation. Namely, (a) \( h^* \to 0 \ (q^* \to 0) \) with \( r_w^* \to 0 \), and (b) \( h^* \to \infty \ (\Delta T^* \to 0) \) with \( r_w^* \to 0 \). For more extensive discussion, see, e.g., Bejan [2].

From the above, it is evident that the ultimate heat transfer technology is to achieve either complete thermal insulation or infinitely large heat transfer rate, while keeping the friction drag minimum. A typical example of the former is gas turbine blade cooling, while that of the latter turbulence promoters in heat exchanger passages. In the present work, we focus on the latter, since this is considered to be much more difficult to achieve due to the inherent similarity between the momentum and heat transfer as discussed below.

Up to the present, there have been made numerous studies to establish correlations for wall skin friction and heat transfer rate in turbulent flows. Reynolds [3] pioneered the concept of analogy between turbulent momentum and heat transfer. This results in the following relationship between the friction coefficient \( C_f \) and the Stanton number \( St \):

\[
St = \frac{C_f}{2}.
\]

The above dimensionless parameters of \( C_f \) and \( St \) are respectively defined as:

\[
C_f = \frac{r_w^*}{\frac{1}{2} \rho^* U_b^*}, \quad St = \frac{C_f}{2}.
\]
\[ \text{St} = \frac{q^*}{\rho^* c_p^* U_b^* T_b^*}, \quad (4) \]

where \( T_b^* \), \( \rho^* \) and \( c_p^* \) are the bulk mean temperature, the fluid density and the specific heat at constant pressure, respectively.

By taking into account the effect of the Prandtl number \( \text{Pr} = \nu^*/\alpha^* \), where \( \nu^* \) and \( \alpha^* \) are the kinematic viscosity and the thermal diffusivity, respectively, Eq. (2) is modified as:

\[ \text{St} = \frac{C_f \text{Pr}^{-2/3}}{2}, \quad (5) \]

which is called Chilton and Colburn Analogy [4], and the most widely accepted concept for predicting turbulent heat and mass transport in practical flows (see, e.g., Kays et al. [5]). Meanwhile, its success suggests inherent difficulty in achieving the dissimilar control of enhancing heat transfer with friction drag reduced or not increased as much as heat transfer. It should be noted, however, this recognition has been reached somewhat intuitively, but not based on rigorous analyses of the governing equations and boundary conditions of momentum and heat transfer.

In general, even though the governing equations of momentum and heat transport appear similar, the dissimilarity between velocity and temperature fields may result from various causes such as dissimilar boundary conditions and different mechanisms of turbulent momentum and heat transfer. Cheng et al. [6] investigated remarkable heat transfer enhancement with the corresponding friction factor reduced in a spirally fluted tube. They found that the bulk swirl motion stabilized turbulence so that the total drag is decreased, while the local recirculation motion in the flute trough as well as the expanded surface area effectively enhance heat transfer. Yabe et al. [7] demonstrated that, by installing an array of wire electrodes near the wall and by applying a high voltage to them to induce local EHD jets, the ratio of the Colburn factor to the friction factor could be increased 4.4 and 1.9 times respectively in the laminar and turbulent oil flow in a rectangular duct. Benhalilou and Kasagi [8] applied micro grooves (riblets) to a turbulent channel flow with heat transfer, and showed that the riblets cause turbulent drag reduction, but the heat transfer is enhanced at high Pr. This is because the thermal boundary layer thickness is thinner than the riblet height, so that the heat transfer surface area is virtually increased around the riblet.
The turbulent heat transfer in a flat plate boundary layer perturbed by a cylinder has been extensively studied by Suzuki et al. [9]. The near-wall turbulence modification due to a blockage effect of the cylinder makes turbulent transport of heat more enhanced than that of momentum. Dissimilarity leading to much different friction coefficient and Stanton number has also been confirmed in the thermal boundary layer with slot injection/suction through direct numerical simulation (DNS) by Kong et al. [10]. They find that the streamwise pressure gradient induced by the abrupt wall injection/suction should be the primary cause of dissimilarity. These pieces of knowledge also suggest that an essential aspect of dissimilar transport mechanisms should be important. Namely, the temperature is a scalar quantity, while the streamwise velocity is one of the three components of a velocity vector; the latter is always subjected to divergence-free condition due to the continuity.

In the last several decades, various successful control strategies have been proposed for turbulent skin friction drag reduction (see, e.g., Kasagi et al. [11, 12], for review). With these controls, the heat transfer characteristics could have been modified. As for simultaneous achievement of skin friction reduction and heat transfer enhancement, however, few studies have been made. An attempt of such a dissimilar control was once made by Yokoo et al. [13] for a turbulent channel flow with isothermal walls kept at different temperatures at a very low Reynolds number. They defined a cost function including the variances of skin friction, wall heat flux, and velocity and temperature fluctuations with their weights, and applied the optimal control procedure to determine the control input, i.e., local blowing/suction from the wall, so that the cost function is minimized. Despite the huge computational cost required for the optimization, the effect of control they found remained very small. Clearly, a different approach with less computational load is desirable.

In the present study, we reexamine the governing equations and boundary conditions from a viewpoint of dissimilarity between the momentum and heat transfer. Based on these analyses, we summarize possible scenarios for dissimilar control in Sec. 2. In Sec. 3, we review the mathematical relationship between the friction drag and the Reynolds stress, and then extend it to heat transfer. In Sec. 4, we will employ the so-called suboptimal control theory in order to make it possible to derive more universal control laws which are applicable even to the situation where the averaged momentum and heat transport equations are apparently identical. Finally, we will summarize the present work in Sec. 5.
2. Possible scenarios of dissimilar control of momentum and heat transfer

2.1. General strategy for dissimilar control

Although the analogy concept was intuitionally proposed [3, 4], it has a mathematical basis under idealized conditions of flow and heat transfer. The governing equations of momentum and heat transport in a Newtonian fluid of constant physical properties are generally given as:

\[
\frac{\partial u_i^*}{\partial t^*} + u_j^* \frac{\partial u_i^*}{\partial x_j^*} = - \frac{1}{\rho^*} \frac{\partial p^*}{\partial x_i^*} + \nu^* \frac{\partial^2 u_i^*}{\partial x_j^* \partial x_j^*},
\]

\[
\frac{\partial T^*}{\partial t^*} + u_j^* \frac{\partial T^*}{\partial x_j^*} = \alpha^* \frac{\partial^2 T^*}{\partial x_j^* \partial x_j^*} + Q^*,
\]

where \( Q \) represents a heat source, which is in general a function of space and time. When deriving Eq. (7) from the original energy conservation equation, the flow velocity is assumed moderate so that we can neglect both the pressure work and the dissipative heat generation. If we assume a two-dimensional flow parallel to a wall, the problem is reduced to solving Eqs. (6) and (7) for the streamwise (tangential) velocity component and scalar temperature. Then, both equations are rewritten as:

\[
\frac{\partial u^*}{\partial t^*} + u^* \frac{\partial u^*}{\partial x^*} + v^* \frac{\partial u^*}{\partial y^*} + w^* \frac{\partial u^*}{\partial z^*} = \frac{1}{\rho^*} \frac{\partial p^*}{\partial x^*} + \nu^* \left( \frac{\partial^2 u^*}{\partial x^* \partial x^*} + \frac{\partial^2 u^*}{\partial y^* \partial y^*} + \frac{\partial^2 u^*}{\partial z^* \partial z^*} \right),
\]

\[
\frac{\partial T^*}{\partial t^*} + u^* \frac{\partial T^*}{\partial x^*} + v^* \frac{\partial T^*}{\partial y^*} + w^* \frac{\partial T^*}{\partial z^*} = \alpha^* \left( \frac{\partial^2 T^*}{\partial x^* \partial x^*} + \frac{\partial^2 T^*}{\partial y^* \partial y^*} + \frac{\partial^2 T^*}{\partial z^* \partial z^*} \right) + Q^*.
\]

It is evident that the above equations are exactly similar if the pressure gradient and heat source terms have the same functional form. In that case, if the solution of \( u \) is obtained for Eq. (8) under similar initial and boundary conditions of two quantities (as discussed later), then the identical solution satisfies Eq. (9). Note that Eqs. (8) and (9) as well as their boundary conditions can be made dimensionless with unique parameters such as Re and Pr, and the equations become completely similar when Pr = 1.

The above fact has been widely exploited for estimating the heat transfer coefficient from the known friction coefficient and vice versa in engineering practice for long time, although the range of validity for this approximate method has not been fully explored. Recently, this similarity has become a focus of fundamental research in a different sense. That is because it is expected to be tremendously beneficial from...
a viewpoint of sustainable energy utilization, if we can develop new technologies for heat/mass transfer enhancement with simultaneous friction loss reduction by conquering this inherent similarity in the transport mechanisms.

The breaking of momentum and heat transfer similarity may arise because of many different causes as in the following cases:

・ The Prandtl number is dependent on a fluid taking typical values on the order of $10^{-2}$ for liquid metals, while 0.7 and 7 for air and water respectively under the standard condition. Viscous fluids such as engine oil have much larger values ranging from $10^2$ to $10^4$. Thus, the departure of Pr from unity makes the governing equations of (8) and (9) themselves dissimilar. At high Pr, the thickness of the thermal boundary layer becomes thinner than that of the momentum boundary layer, so that the flow and thermal fields become much different.

・ The Prandtl number significantly affects the turbulent heat transport mechanism, which may cause significant dissimilarity. Numerous studies have been made to obtain the turbulent Prandtl number $Pr_t = \frac{\nu_t}{\alpha_t}$. The value of $Pr_t$ is dependent upon $Pr$ as well as Re, particularly near a wall (see, e.g., Launder [14]; Myong et al. [15]). Shaw and Hanratty [16] show that the temperature fluctuation inside the viscous sublayer is governed by lower frequencies at high Prandtl numbers. This low-pass filtering effect leads to the drastic increase of the turbulent Prandtl number near the wall [17]. As for low Pr, numerical simulation studies have been made by, e.g., Kim and Moin [18], Kasagi and Ohtsubo [19], and Kawamura et al. [20]. They show that the turbulent Prandtl number deviates from unity throughout the channel.

・ There is no similarity when arbitrary body force or heat source appears in the governing equations. For instance, when the buoyancy or MHD force drives or affects the flow, we cannot expect the thermal field to closely resemble the flow field.

・ Unlike the “no-slip” (Dirichlet) condition for the velocity, there exist a variety of thermal boundary conditions at a wall. In reality, such interfacial thermal conditions can only be determined by the conjugate heat transfer coupled with heat conduction inside a wall [21]. For the no-slip boundary condition, the similarity is expected to hold only for isothermal wall condition with no temperature fluctuation. Note that the thermal field with an arbitrary wall temperature distribution can be composed by the superposition of multiple solutions for the energy equation with step temperature distributions.
(see, e.g., Kays et al. [5]), but the similarity cannot generally be expected in this case. The constant heat flux (Neumann) condition is clearly not similar to the no-slip condition.

- The velocity fluctuation is usually under the no-slip boundary condition, but this is not necessarily the case for a permeable wall and when blowing or suction is made on a wall. Likewise, temperature fluctuation may arise independently from velocity fluctuation in some active flow control.

From the argument above, the basic strategies for achieving dissimilar control of momentum and heat transfer in turbulent flow is generally classified as follows:

[A] In terms of the averaged quantities:

1a) To introduce a dissimilar source term in the momentum or energy equation.

1b) To exploit the Prandtl number effect, which makes momentum and thermal boundary layer thicknesses different.

2) To impose dissimilar boundary conditions.

[B] In terms of the fluctuating quantities (turbulent stress and flux):

3a) To exploit the difference between divergence-free vector and conservative scalar quantities.

3b) To exploit the Pr effect on scalar fluctuations.

4) To introduce dissimilar boundary conditions for velocity ($u'$) and temperature ($T'$) fluctuations.

Although many existing technologies of convective heat transfer enhancement fall in the categories above, there is still room for possible improvement. On the other hand, the strategies categorized as [B](3) and [B](4) have not been well recognized and tested so far. In the following sections, we discuss these strategies in more details by examining specific examples.

2.2. Dissimilarity in averaged equations

We consider a fully developed turbulent channel flow as shown in Fig. 1. The streamwise, wall-normal and spanwise directions are denoted as $x$, $y$ and $z$, respectively, where the origin of $y$ is located at the
bottom wall. In the following, all quantities are normalized by the channel half width $\delta^*$ and the bulk mean velocity $U_b^*$. The averaged streamwise momentum balance reduces to

$$0 = -\frac{\partial}{\partial y} \left( \overline{u''v'} - \frac{2}{\text{Re}_b} \frac{\partial \overline{u'}}{\partial y} \right) + \overline{F}.$$  \hspace{1cm} (10)

Here, a variable with an over-bar represents a value averaged in the homogeneous directions, \textit{i.e.}, $x$ and $z$, and also time $t$, while a quantity with a prime denotes a fluctuation component. Note that the mean pressure gradient is denoted as the mean force term $\overline{F}$ in Eq. (10) and the factor of 2 in the viscous term appears according to the definition of bulk Reynolds number $\text{Re}_b = U_b^* (2\delta^* \nu^*)$. The no-slip condition at the walls is described as:

$$\Pi = 0 \quad \text{at} \quad y = 0 \quad \text{and} \quad 2.$$ \hspace{1cm} (11)

Similarly, the mean energy transport equation is given as:

$$0 = -\frac{\partial}{\partial y} \left( \overline{\theta''v'} - \frac{2}{\text{Pr} \text{Re}_b} \frac{\partial \overline{\theta'}}{\partial y} \right) + \overline{Q}.$$ \hspace{1cm} (12)

The dimensionless temperature $\theta$ is normalized by an appropriate temperature scale in the problem under study. From Eqs. (10) and (12), it is evident that the molecular transport terms are different when the Prandtl number is not unity, so the departure from the Reynolds analogy, \textit{viz.}, dissimilar control would be possible at both low and high Prandtl numbers.

The mean pressure gradient term in Eq. (10) is uniform throughout the channel in a fully developed flow, whereas this is generally not true for the heat source term $\overline{Q}$. For instance, if there is uniform heat generation in the fluid (like chemical reaction, say), then $\overline{Q}$ is constant [18], while $\overline{Q}$ is proportional to the local mean velocity when the fluid is heated (or cooled) at a constant heat flux from the two walls [22]. This difference between the source terms, $\overline{F}$ and $\overline{Q}$, in Eqs. (10) and (12) may give a chance of dissimilar heat transfer enhancement.

As for the thermal boundary conditions, two extremes are often considered in theoretical analyses, \textit{i.e.}, constant wall temperature difference (CTD) and constant wall heat flux (CHF) conditions. Conceptual temperature profiles under these conditions are illustrated as $\theta_t$ and $\theta_h$ in Fig. 1. These different thermal boundary conditions give different forms of $\overline{Q}$ in Eq. (12). Therefore, in most engineering flows, the
governing equations and the wall boundary conditions for $\bar{u}$ and $\bar{\theta}$ are different. This fact results in different turbulence contributions to the skin friction and the heat flux at the wall. In Sec. 3, we consider the above two typical thermal conditions, and under each condition examine the contribution of laminar and turbulent transport mechanisms to the wall heat flux. Such insight provides useful strategies for dissimilar control.

2.3. Dissimilarity in fluctuation equations

Consider the case where Eqs. (10) and (12) take an identical form, i.e.,

$$\bar{Q} = \bar{F} = -\frac{\partial \bar{p}}{\partial x} \quad \text{and} \quad \text{Pr} = 1.0.$$ (13)

Physically, the former condition corresponds to uniform heat generation in the fluid. In addition, there is a case where we can define the wall thermal boundary condition as $\bar{\theta} = 0$ at both walls, which is similar to the Dirichlet condition of Eq. (11). Then, both the governing equations and the boundary conditions for the mean streamwise velocity and temperature become exactly the same so that one may consider dissimilar control is even more difficult to achieve than in other cases. However, the dissimilar control is still possible, since the fluctuating components of $u'_i$ and $\theta'$ appearing in the Reynolds stress and turbulent heat flux terms in Eqs. (10) and (12) have different characteristics as shown below.

In order to examine the above argument, we revisit the transport equation of $u'_i$ given as:

$$\frac{\partial u'_i}{\partial t} + \frac{\partial u'_j}{\partial x_j} + u'_i \frac{\partial \bar{u}_i}{\partial x_j} = -\frac{\partial \bar{p}/\partial x_i}{\partial x_j} + \frac{\partial}{\partial x_j} \left( u'_i u'_j - u'_i u'_j \right) + 2 \text{Re}_k \frac{\partial^2 u'_i}{\partial x_j \partial x_j}. \quad (14)$$

The transport equation for $\theta$ can also be obtained by replacing $u'_i$ with $\theta'$ except for the pressure gradient term. In an incompressible fluid, the pressure fluctuation instantaneously responds to the fluctuating velocity field so as to project it to the divergence-free space. Therefore, the three components of a velocity vector are coupled through the pressure gradient term. On the other hand, a passive scalar
quantity does not have such restriction. This fundamental difference should affect the dynamics of $u'$ and $\theta'$ even in an uncontrolled flow.

In Sec. 4, we make an attempt to establish a control algorithm for turbulent heat transfer enhancement by exploiting such inherent dissimilarity between the velocity and scalar quantities in the framework of the suboptimal control theory.

3. Dissimilar control strategy based on identity equations

3.1. Mathematical derivation

In this section, we discuss the strategy categorized as [A] mentioned in Section 2.1. Fukagata et al. [23] has derived the general mathematical relationship, now called FIK identity, between the skin friction coefficient $C_f$ and different dynamical contributions in wall-bounded turbulent flows. It is simplified to the following equation including two terms in the case of a fully developed turbulent channel flow:

$$C_f = \frac{12}{Re_b} + 6 \int_0^1 (1 - y) \left(-\overline{u'v'}\right)dy.$$  \hspace{1cm} (15)

The first term in the RHS is the laminar drag and the second term is the additional friction in the turbulent flow. The Reynolds shear stress, which is weighted by the distance from the wall $(1-y)$ in the second term of RHS of Eq. (15), implies that reduction of Reynolds shear stress near the wall is more effective for friction drag reduction than that far from the wall. Note that the coefficient for the second term is different from the original [23] because of different nondimensionalization. This identity equation has been widely used for quantifying the control effect as well as developing new control methods (see, Kasagi et al. [24, 25] for review).

Similar decomposition, but for the wall heat flux may be useful in order to design the control of heat transfer. In this section, we extend the recent work made by Fukagata et al. [26]. First, an explicit mathematical relationship is derived of laminar and turbulent contributions to the wall heat flux in a fully
developed turbulent channel flow, and then the control strategy derived thereof is examined by means of numerical simulation.

Three different thermal boundary conditions are considered: 1) constant, but different temperatures on two walls – constant temperature difference (Case CTD), 2) constant heat flux on two walls (Case CHF), and 3) uniform heat generation in fluid between two isothermal walls (UHG). In all cases, temperature fluctuation is assumed to be zero on the walls. In Case CTD, one wall is heated and the other is cooled, so that they are kept at given (different) temperatures. The problem is to obtain larger wall heat flux under the given temperature difference. In Case CHF, both walls are heated with a prescribed wall heat flux. The temperature difference between the fluid and the wall, which is determined as a result of turbulent heat transport, should be reduced. In Case UHG, there exists uniform heat generation (or absorption) in the fluid, while the wall temperatures are kept constant. From the global heat balance, one finds that the wall heat flux is equal to the total heat generation, \( i.e., \ \left( \frac{\partial \theta}{\partial y} \right)_{w=0} = \frac{\text{Pr} \cdot \text{Re}_b \cdot Q}{2} \). Namely, similar to CHF case, the temperature difference is determined as a result of the effectiveness of turbulent heat transport. Due to this difference in the problem setting, the variables used are made dimensionless with different characteristic quantities in each case. Therefore, the derivation process for each case is separately presented below.

**Case CTD: Constant temperature difference**

By assuming no heat source, \( i.e., \ \bar{Q} = 0 \) in Eq. (12), the transport equation for the dimensionless mean temperature reads

\[
0 = -\frac{\partial \left( \theta' \nu' \right)}{\partial y} + \frac{2}{\text{Pr} \cdot \text{Re}_b} \frac{\partial^2 \bar{\theta}}{\partial y^2},
\]

where the dimensionless temperature \( \theta \) is defined by using a half of the temperature difference between the top and bottom walls \( \theta = \left( T^\ast - T^\ast \right) / \Delta T^\ast \) with \( \Delta T^\ast = \left( \left| T^\ast \right|_{y=2} - \left| T^\ast \right|_{y=0} \right) \) and \( \Delta T^\ast = \left( \left| T^\ast \right|_{y=2} - \left| T^\ast \right|_{y=0} \right) \). By applying double integration to Eq. (16), we obtain
The above Stanton number is defined as \( \text{St} = \frac{\text{Nu}}{\text{Pr Re}_b} \) with the Nusselt number, 
\( \text{Nu} = \frac{(2\delta^*) q^*}{(\lambda^* \Delta T^*)} \),
in which \( \lambda^* \) and \( q^* \) are the thermal conductivity and the wall heat flux, respectively.

The above equation suggests that the Stanton number can be decomposed into two parts. The first term on the RHS of Eq. (17) is the laminar contribution, which is identical to the conductive transport, whilst the second term is the turbulent transport. The latter is a simple integration of turbulent heat flux \((-\sqrt{\theta'})\) unlike the turbulent contribution to the skin friction coefficient in Eq. (15), where the Reynolds shear stress at different location contributes to the wall skin friction to a different degree.

**Case CHF: Constant heat flux**

In this case, the constant wall heat flux, \( q_w^* \), is given. Therefore, we introduce another dimensionless temperature, \( \theta \) (which is different from \( \theta \) above), i.e., \( \theta = (T_w^* - T^*) / \Delta T_x^* \), where \( T_w^* (x) \) is the wall temperature linearly varying in the streamwise direction. From the global heat balance, the reference temperature of \( \Delta T_x^* \) is found to be equal to the change of \( T_w^* \) and also of the bulk mean temperature, \( T_b^* \) over the streamwise distance of \( \delta^* \), i.e.,

\[
\Delta T_x^* = \delta^* \frac{dT_w^*}{dx} = \frac{q^*}{\rho c_p \overline{U}_b}.
\]  

(18)

By using Eq. (18), the transport equation for the dimensionless mean temperature reads (see also Kasagi et al., [22])

\[
0 = -\frac{\partial \overline{\theta'}}{\partial y} + \frac{2}{\text{Pr Re}_b} \frac{\partial^2 \overline{\theta'}}{\partial y^2} + \frac{u}{\overline{U}_b}.
\]  

(19)

Note that the above equation corresponds to \( \overline{Q} = \overline{u} \) in Eq. (12). By applying triple integration to Eq. (19), we obtain the expression for the inverse of Stanton number:

\[
\text{St} = \frac{2}{\text{Pr Re}_b} + \int_0^1 (-\sqrt{\theta'}) \, dy.
\]  

(17)
\[
\frac{1}{St} = \operatorname{Pr} \operatorname{Re}_b \left[ \frac{17}{70} - \frac{1}{2} \int_0^1 \left( 1 - \phi \right) \left( -v' \theta' \right) dy - \frac{1}{2} \int_0^1 \left( v' - 3 y' + 1 \right) \phi_x - \phi_d \right] dy + \frac{1}{2} \int_0^1 u' \theta' \ dy, \tag{20}
\]

where the two functions of \( \phi(y) \) and \( \phi_d(y) \) are defined as:

\[
\phi(y) = \int_0^y \pi(\eta) d\eta, \quad \phi_d(y) = \int_0^y \pi_d(\eta) d\eta, \tag{21}
\]

with \( \pi_d \) being the deviation of the mean velocity from the laminar profile, which causes a change in the bulk mean temperature.

The first term in the bracket on the RHS of Eq. (20) corresponds to the heat transfer in a laminar channel flow with isoflux walls, \( i.e., St \operatorname{Pr} \operatorname{Re}_b = Nu = 70/17 \), twice of which \( i.e., \) based on the hydraulic diameter is a well known value, \( Nu_d = 2Nu = 140/17 = 8.235 \) (see, \( e.g., Kays et al., [5] \)). The second term represents the contribution by the turbulent heat flux, and is usually positive in a turbulent channel flow. This term results in reduction of \( 1/St \) \( i.e., \) increase of \( St \) compared to that of laminar flow. The third and fourth terms reflect the modification of bulk mean temperature due to the deviation of mean velocity profile from the laminar Poiseuille profile and the additional enthalpy flux due to the streamwise turbulent heat flux, respectively. According to DNS data, these two terms are found to be minor as compared to the second term.

**Case UHG: Uniform heat generation**

The transport equation for the mean temperature is given by Eq. (12), where the heat source,

\[
\dot{Q} = \frac{2}{\operatorname{Pr} \operatorname{Re}_b} \left. \frac{\partial \theta}{\partial y} \right|_{y=0}, \tag{22}
\]

is uniform in the channel. The temperature is made dimensionless so that \( \bar{\theta} = 0 \) on the wall. The derivation is essentially similar to Case CHF and the identity equation for the Stanton number is derived as:

\[
\frac{1}{St} = \operatorname{Pr} \operatorname{Re}_b \left[ \frac{1}{5} - \frac{1}{2 \dot{Q}} \int_0^1 \left( 1 - \phi \right) \left( -v' \theta' \right) dy - \frac{1}{\dot{Q}} \int_0^1 \left( 1 - \theta \right) \phi_d \ dy \right] \frac{1}{2} \int_0^1 u' \theta' \ dy. \tag{23}
\]

Equation (23) indicates that the weighting factor for the contribution from turbulent heat flux is similar to that in Case CHF, although the transport equation of heat in this case takes a similar form to that of momentum as mentioned in Sec. 2.3.
3.2. Assessment of proposed control laws

In the case of isothermal walls at different temperatures (CTD), the different weighting functions of turbulent contribution terms for the skin friction in Eq. (15) and for the heat transfer in Eq. (17), i.e., \((1-y)\) and 1, suggest that simultaneous control of drag reduction and heat transfer augmentation may be possible, if the turbulence is suppressed near the wall and enhanced in the central region of the channel.

In the case of constant heat flux heating from both walls (CHF), the derived relationship indicates that achievement of simultaneous friction drag reduction and heat transfer augmentation is difficult because the weighting function of \((1-\phi)\) has a distribution quite similar to that for the skin friction, i.e., \((1-y)\).

The proposed strategy is examined by means of DNS of a fully developed turbulent channel flow at \(Re_b = 3220\) and \(Pr = 0.71\). This Reynolds number corresponds to the friction Reynolds number, \(Re_c = 110\), which is defined based on the friction velocity and the channel half width. The DNS code is based on the pseudo-spectral method, and the computational details are found in Iwamoto et al. [27]. The opposition control scheme [28] is employed to suppress the near-wall Reynolds stress. The virtual detection plane for sensing the wall-normal velocity component is located at \(y_d^+ = 10\) in order to reduce the Reynolds shear stress near the wall. In addition, a virtual body force is assumed in the wall-normal momentum equation for the enhancement of turbulent heat-flux in the central region of the channel. It is given as \(-bf(y)\theta'\), where \(b\) is the magnitude and \(f(y)\) is the envelope function. The latter defines the region where the body force works. This ad-hoc term appears as an additional source term in the transport equation of \(\theta\) as to enhance the heat transport in proportion to the instantaneous temperature at \(y_d^+ = 10\).

Here, Case CTD is examined. According to the strategy above, the envelope function is set to have a value of unity in the central region away from the wall and zero near the wall: \(f(y) = 1\) for \(0.5 < y < 1.5; f(y) = 0\) for \(0 < y < 0.5\) and \(1.5 < y < 2\). The amplitude coefficient has been changed, but the result at \(b^+ = 0.01\) is shown below. Figures 2 (a) and (b) show the time traces of the skin friction coefficient, \(C_f\), and the Stanton number, \(St\), respectively, where three cases are represented, i.e., (a) no control, (b) the opposition control (denoted as w/V-control), and (c) the opposition control combined with the body force. In the case of (b), both \(C_f\) and \(St\) start to decrease immediately after the onset of control and later remain at much smaller levels compared to the case without control. With the combined control of (c), both quantities...
initially follow the traces with the opposition control, but later much increase. After the initial transient, $CF$ returns to the level of the uncontrolled flow, whilst $St$ further increases beyond the original level to about 1.5 times. The profiles of Reynolds shear stress and turbulent heat flux are changed as we expected as shown in Fig. 3. The Reynolds shear stress profile is almost the same as that in the uncontrolled case. The turbulent heat flux is also suppressed near the wall ($y < 0.1$), whereas it is largely enhanced in the central region of the channel because of the body force introduced at $0.5 < y < 1.5$. This change leads to significant increase in the heat transfer as implied by Eq. (17).

4. Suboptimal control strategy

4.1. Derivation of control law

In the previous section, we discussed possibilities of achieving dissimilar control (Category [A]) based on the averaged momentum and heat transfer equations (10) and (12). Such strategies, however, turns out to be limited to the cases where the averaged momentum and heat transport equations have dissimilar forms. In this section, we pursue the strategy of Category [B] by employ the suboptimal control theory to derive a more universal control scheme applicable to the cases where the averaged momentum and heat transport equations are similar [29]. We assume $Q = -\left(\bar{p}/\partial x\right) = const.$ and $Pr = 1.0$. Consequently, the averaged momentum and heat transport equations have the identical form (see, Sec. 2.3).

In the present study, we consider local wall blowing/suction as a control input. Since the control surface area is assumed sufficiently large, there is not net mass flux across the walls. As for the tangential velocity components and the temperature, we impose the no-slip and isothermal conditions. The resultant boundary conditions are described as:

\begin{align}
    u_i &= \phi_B \delta_{i2}, \quad \theta = 0 \quad at \quad y = 0, \\
    u_i &= \phi_T \delta_{i2}, \quad \theta = 0 \quad at \quad y = 2,
\end{align}

(24)  

(25)

where $\phi_B$ and $\phi_T$ represent the control inputs (given velocities) at the bottom and top walls, respectively. Note that the boundary conditions for $u_l$ and $\theta$ remain similar even in the controlled flow.
Following Lee et al. [30], we discretize the Navier-Stokes and energy equations so that the diffusion and pressure gradient terms are treated implicitly, while the advection terms explicitly. This results in the following equations representing the short-time evolution of the system:

\[
\begin{align*}
  u_i^{n+1} + \frac{\Delta t}{2} \frac{\partial p^{n+1}}{\partial x} - \frac{\Delta t}{Re_b} \frac{\partial^2 u_i^{n+1}}{\partial x \partial x_j} &= R^n, \quad (26) \\
  \frac{\partial u_i^{n+1}}{\partial x_j} &= 0, \quad (27) \\
  \theta^{n+1} - \frac{\Delta t}{Pr Re_b} \frac{\partial^2 \theta^{n+1}}{\partial x \partial x_j} &= Q^n. \quad (28)
\end{align*}
\]

Here, the superscripts \( n \) and \( n+1 \) represent successive time steps. By introducing a Fréchet differential, the differential states of the velocity, pressure and temperature are defined as:

\[
\begin{align*}
  \xi_i &= \frac{Du_i^{n+1}(\phi)}{D\phi} \tilde{\phi}, \quad (29) \\
  \sigma &= \frac{Dp^{n+1}(\phi)}{D\phi} \tilde{\phi}, \quad (30) \\
  \eta &= \frac{D\theta^{n+1}(\phi)}{D\phi} \tilde{\phi}. \quad (31)
\end{align*}
\]

In the above, \( \tilde{\phi} \) is an arbitrary perturbation to the control input \( \phi \). Applying the Fréchet differential to Eqs. (26)-(28), the governing equations for differential states \( (\xi_i, \sigma, \eta) \) are obtained as:

\[
\begin{align*}
  \xi_i + \frac{\Delta \tau}{2} \frac{\partial \sigma}{\partial x} - \frac{\Delta t}{Re_b} \frac{\partial^2 \xi_i}{\partial x \partial x_j} &= 0, \quad (32) \\
  \frac{\partial \xi_i}{\partial x_j} &= 0, \quad (33) \\
  \eta - \frac{\Delta t}{Pr Re_b} \frac{\partial^2 \eta}{\partial x \partial x_j} &= 0. \quad (34)
\end{align*}
\]
The boundary conditions for the differential states are

\[ \xi_i = \hat{\varphi}_i \delta_{ij}, \quad \eta = 0 \quad \text{at} \quad y = 0, \]  
(35)

\[ \xi_i = \hat{\varphi}_i \delta_{ij}, \quad \eta = 0 \quad \text{at} \quad y = 2. \]  
(36)

Solving Eqs. (32)-(34) under the boundary conditions of Eqs. (35) and (36), we eventually obtain the following approximate solutions:

\[ \hat{z}_i = \frac{ik_z}{k} \hat{\varphi} \left\{ \exp(-ky) + \exp \left( -\frac{\text{Re}_{\alpha}}{k^2} y \right) \right\}, \]  
(37)

\[ \hat{z}_3 = \frac{ik_z}{k} \hat{\varphi} \left\{ \exp(-ky) + \exp \left( -\frac{\text{Re}_{\alpha}}{k^2} y \right) \right\}, \]  
(38)

\[ \hat{z}_2 = \hat{\varphi} \exp(-ky), \]  
(39)

\[ \hat{\sigma} = \frac{2}{k} \hat{\varphi}, \]  
(40)

\[ \hat{\eta} = 0. \]  
(41)

Equations (37)-(41) represent the responses of the velocity and scalar fields to an infinitesimal change of the control input. It is worth noting that the obtained differential states for the streamwise velocity and temperature are different. When the wall blowing/suction is applied, the pressure field instantaneously reacts so as to redistribute the kinetic energy of the wall-normal velocity fluctuation to the tangential directions. In the case of the scalar field, however, it does not possess such mechanism, and therefore
\( \dot{\eta} = 0 \). This fact shows an essential difference between the velocity and scalar fields, and suggests a possibility of dissimilar control.

In order to achieve dissimilar control of momentum and heat transfer, we define the cost function \( J \) as follows:

\[
J = \frac{1}{S \Delta t} \int_{t_i}^{t_{i+1}} \left( \frac{1}{2} (\dot{\phi}^2 + \dot{\psi}^2) ds + \frac{\beta}{V \Delta t} \int_{t_i}^{t_{i+1}} (1-y)(-u'_1 u'_2) dtdV - \frac{\gamma}{V \Delta t} \int_{t_i}^{t_{i+1}} (1-y)(-\partial u'_4) dtdV \right), \tag{42}
\]

where the temporal integration is made over a short computational time step \( \Delta t \). The spatial integration is also made over the wall surface \( S \) for the first term, while over the whole flow domain \( V \) for the second and third terms. Our goal is to deduce the optimal spatiotemporal distribution of control input \( \phi \) to minimize \( J \).

The first term in Eq. (42) represents the cost of actuation \( \phi \). In accordance with Eqs. (15) and (23), we introduce the weighted Reynolds stress and weighted turbulent heat flux to the second and third terms in order to evaluate the friction drag and heat transfer, respectively. The coefficients of \( \beta \) and \( \gamma \) denote the measures of relative costs (or merits) of friction drag and heat transfer against the control input, respectively. We give a negative sign to the third term so as to seek the least control input that enhances heat transfer while reducing the friction drag.

Applying the Fréchet differential to Eq. (42) and substituting Eqs. (37)-(41), the optimal control input is obtained as:

\[
\hat{\phi} = \int_{y'} e^{-y'} (1-y') \left[ \beta \left( \hat{u}(y') + \frac{ik}{k} \hat{v}(y') \right) - \gamma \hat{\theta}(y') \right] dy', \tag{43}
\]

where \( y' \) represents the distance from the wall. Again, we assume that \( \text{Re}_b / \Delta t >> k^2 \) in driving Eq. (43).

\subsection*{4.2. Dissimilar heat transfer enhancement}

The computation is made under the condition of a constant bulk mean velocity \( U_b^* \), which results in the bulk mean Reynolds number of \( \text{Re}_b = 4560 \). The blowing/suction is applied at both walls. The magnitude of this control input normalized by \( U_b^* \) is kept constant during each calculation so that only the ratio of \( \beta \) and
\( \gamma \) in Eq. (43) is of major concern. In the following, we introduce typical results when \((\beta, \gamma) = (1, 1)\). The results obtained with different combinations of \( \alpha \) are reported in Hasegawa & Kasagi [29].

Figure 4 shows the friction coefficient \( C_f \) and the Stanton number \( S_t \) as a function of time from the onset of control. With increasing \( \phi_{rms} \), both \( C_f \) and \( S_t \) first increase and then abruptly decrease at \( \phi_{rms}/U_b = 0.1 \). In Fig. 5, the effect of the magnitude of \( \phi_{rms} \) on the normalized \( j/f \) factor is shown. It is increased up to about 1.5 with increasing \( \phi_{rms} \), and then saturates when \( \phi_{rms}/U_b > 0.05 \). Hence, for both the heat transfer enhancement and the \( j/f \) factor, there exists an optimal magnitude, which is estimated to be about 5% of the bulk mean velocity under the present condition.

Instantaneous distributions of the control inputs on the top and bottom walls are shown in Figs. 6(a) and (b), respectively. It is obvious that the control input is distributed almost uniform in the spanwise direction, while periodic in the streamwise direction. This pattern is found to travel in the downstream direction at an almost constant speed, i.e., around 30% of the bulk mean velocity. It is also found in Fig. 6 that the control input is introduced in a varicose-mode. When \( v_w \) is positive (blowing) on the bottom wall, negative \( v_w \) (also blowing) is found on the opposite top wall. Note that the blowing occurs more strongly in a narrower region than the suction. Kong et al. [10] carried out numerical simulation of turbulent boundary layer disturbed by blowing/suction through a spanwise slot provided on the wall and also found that the blowing leads to stronger dissimilarity than the suction. It has been also reported that streamwise-periodic wall blowing and suction in the form of an upstream traveling wave causes drag reduction in turbulent channel flow at a low Reynolds number [31]. This drag reduction is primary due to the pumping effect in the direction opposite to the traveling wave [32], which in turn suggests that the drag is increased with a wave traveling downstream. In the present suboptimal control, the drag is in fact increased with a downstream traveling wave-like control, but the heat transfer is more enhanced, and therefore the dissimilar heat transfer enhancement is made possible (see, Figs. 4 and 5). Considering the fact that the pumping effect by streamwise traveling wave is caused by a phase difference between the fluctuating streamwise and wall-normal velocity components [33], there should be a notable phase difference between the fluctuating streamwise velocity and temperature in this dissimilar case.

Instantaneous snap-shots of the streamwise velocity and temperature respectively normalized by \( U_b \) and \( \Theta_b \) in the \( x-y \) plane are shown in Fig. 7(a) and (b), respectively. Note that they are taken at the same instance as in Fig. 6. It is confirmed that the responses of the streamwise velocity and the temperature to the
present control input are totally different. More specifically, just downstream of the region with wall blowing around \( x = 3.0 \), the streamwise velocity is strongly accelerated due to the contraction effect caused by blowing, although blowing itself introduces low momentum fluid into the flow field. On the other hand, the bulk mean temperature in this region becomes lower. This is because low temperature \( (\theta = 0) \) fluid is injected from the wall but the fluid temperature in the central part of the channel is not immediately affected by blowing. It should be noted that the substantial acceleration of \( u' \) downstream of wall blowing is essentially driven by the favorable streamwise pressure gradient, which appears only in the momentum transport equation. The present results clearly demonstrate that the vector and scalar quantities are essentially different in their responses to control input introduced, and such inherent dissimilarity is a key to achieve dissimilar heat transfer enhancement. More detailed mechanisms of dissimilarity caused by the traveling wave-like control input are discussed in Hasegawa and Kasagi [29].

5. Conclusions

In order to advance fundamental knowledge and establish basic strategy for better design of thermo-fluid systems such as heat exchangers, power plants and production processes, we have focused upon turbulent heat transfer as one of the most important interfacial transport phenomena. This classical issue poses a very difficult task once we try to develop a methodology of enhancing heat transfer with skin friction reduced or at least not increased as much as heat transfer. This fact is easy to understand if we remind that the turbulence itself is regarded as a nonlinear dissipative dynamical system, which often exhibits very complex behavior and characteristics. Nevertheless, the recent progress in analyzing turbulence mechanics and designing turbulence control offers a chance for us to revisit the problem of dissimilar momentum and heat transport.

By reexamining the governing equations and boundary conditions for a simple parallel flow and its convective heat transfer, the basic strategies for achieving dissimilar control in turbulent flow are generally classified into two groups, i.e., one for the averaged quantities and the other is for the fluctuating turbulent components. For the former, mentioned are three schemes introducing different source terms, Prandtl number far from unity, and dissimilar thermal boundary condition, whilst for the latter exploiting the inherent difference in vector \textit{versus} scalar quantities, their fluctuating component spectra, and boundary
conditions. Since we want to clarify an essential aspect of the problem, we have limited our analysis to a canonical system, i.e., a fully developed turbulent channel flow with heat transport across solid walls under different thermal conditions. As a result, we propose two distinct approaches.

The first is to exploit the universal relationship between skin friction and Reynolds stress in a wall-bounded flow under consideration (FIK identity), and extend it to the heat transfer under the constant wall-temperature difference (CTD), constant wall heat flux (CHF) and uniform heat generation (UHG) conditions. In these mathematical relationships, the contribution of turbulence to the wall shear stress and the wall heat flux appears with different weighting functions of the distance from the wall. They are also different in the three cases of thermal boundary conditions considered, CTD, CHF and UHG. These differences in the way how the local turbulent transport of momentum and heat contributes to the friction and heat transfer coefficients are keys to answer whether the dissimilar control of turbulent heat transfer is feasible or not. The present result shows that the dissimilar control is likely to be achieved when the weight distributions for the stress and flux are different due to the thermal boundary condition like CTD and/or the Prandtl number far from unity.

To overcome the above difficulty with a more general methodology, we introduce the optimal control theory as the second approach. The Fréchet differentials obtained (Eqs. (37)-(41)) clearly show that the responses of the velocity and scalar fields to the wall blowing/suction are quite different. This is due to the fact that the velocity is a dissipative vector and always under the restriction of continuity while the temperature is a conservative scalar without such restriction. By exploiting this inherent difference, the dissimilar control can be achieved even in flow and heat transfer problems where the averaged momentum and heat transport equations have the same form. The present computation is limited, and further study should be made to fully explore the possibility of this approach.

Although the present study demonstrates possible dissimilar turbulent transport controls, the two approaches above require the state information of velocity and temperature in the whole flow domain in order to determine the control input. Toward practical applications, it is required to establish feedback control laws based on the wall information only or predetermined control schemes, which do not need any state feedback. Clearly, the ultimate goal is to develop passive devices that offer equivalent control performance as active controls.
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Nomenclature

\( C_f \)  friction coefficient, \( \frac{1}{2} \rho \frac{u'^2}{\delta} \)

\( c_p \)  specific heat at constant pressure

\( F \)  force

\( f \)  friction factor, \( 4C_f \)

\( h \)  heat transfer coefficient

\( J \)  cost function

\( j \)  factor, \( \frac{Nu}{RePr^{1/3}} \)

\( k \)  wave number

\( \text{Nu} \)  Nusselt number, \( 2h'\delta/\lambda' \)

\( \text{Pr} \)  Prandtl number, \( \nu'/\alpha' \)

\( \text{Pr}_t \)  turbulent Prandtl number, \( v_t'/\alpha_t' \)

\( p \)  pressure

\( Q \)  heat source

\( q \)  heat flux

\( q_i \)  Fréchet differential of \( i \)-th velocity component

\( \text{Re} \)  Reynolds number

\( \text{Re}_b \)  bulk mean Reynolds number, \( 2u_b'\delta/\nu' \)

\( \text{Re}_t \)  friction Reynolds number, \( \frac{u'\delta}{\nu'} \)

\( S \)  surface area

\( \text{St} \)  Stanton number, \( q' / \rho c_p T_b' \)

\( s \)  entropy

\( T \)  temperature

\( T_b' \)  bulk mean temperature, \( \frac{1}{2u_b'\delta} \int_0^{2\delta} uT dy \)

\( t \)  time

\( U_b' \)  bulk mean velocity, \( \frac{1}{2\delta} \int_0^{2\delta} u' dy' \)
\( u \)  

streamwise velocity

\( u_i \)  

velocity component in \( i \)-th direction

\( V \)  

volume

\( v \)  

wall-normal velocity

\( W \)  

work

\( w \)  

spanwise velocity

\( x \)  

streamwise coordinate

\( x_i \)  

coordinate in \( i \)-th direction

\( y \)  

wall-normal coordinate

\( z \)  

spanwise coordinate

**Greek**

\( \alpha \)  

thermal diffusivity

\( \beta \)  

relative price of drag reduction

\( \Delta \)  

difference or increment

\( \delta \)  

channel half width

\( \varepsilon \)  

dissipation rate

\( \gamma \)  

relative price of heat transfer enhancement

\( \phi \)  

function in Eq. (21) or control input in Eqs. (35)-(37)

\( \eta \)  

Fréchet differential of temperature

\( \lambda \)  

thermal conductivity

\( \nu \)  

kinematic viscosity

\( \Theta_b \)  

dimensionless bulk mean temperature

\( \theta \)  

dimensionless temperature, \( T^*/\Delta T^* \)

\( \rho \)  

density

\( \sigma \)  

Fréchet differential of pressure

\( \tau \)  

shear stress

\( \xi \)  

Fréchet differential of \( i \)-th velocity component
\( \chi \) function in Eq. (24)

**Superscript**

\( \ast \) dimensional quantity

\( \dagger \) dimensionless quantity in wall unit

\( \ddagger \) fluctuation

\( \overline{} \) average

\( \tilde{} \) perturbation

\( \hat{} \) Fourier coefficient

**Subscript**

\( b \) bulk mean

\( f \) fluid

\( p \) pumping

\( t \) turbulent

\( w \) at a wall

\( o \) uncontrolled flow
References


Footnotes

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Captions for Figures

Fig. 1: Coordinate system and mean velocity and temperature distributions ($u$: streamwise velocity, and $\theta_A$ and $\theta_B$: temperature in constant temperature difference (CTD) and constant heat flux (CHF) conditions, respectively)

Fig. 2: Time traces of (a) $C_f$, (b) $St$, and (c) $j/f$ factor in Case CTD ($b^+ = 0.01$).

Fig. 3: Turbulence statistics in Case CTD ($b^+ = 0.01$): (a) Reynolds shear stress; (b) turbulent heat flux.

Fig. 4: Time traces of friction coefficient $C_f$ and Stanton number $St$ with different magnitudes of control input $\phi$.

Fig. 5: Time traces of $j/f$ factor with different magnitudes of control input $\phi$.

Fig. 6: Instantaneous distributions of the control input $\phi$ (red contours: wall blowing; blue contours: wall suction): (a) bottom wall, (b) top wall.

Fig. 7: Instantaneous distributions of (a) streamwise velocity and (b) temperature in the $x$-$y$ plane at $z = 1.5$. 
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