Numerical Simulation of Flow Around Two Airfoils in Formation Flight

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Wing-tip vortices are known to deteriorate the airfoil performance due to the induced drag. However, birds utilize the wing-tip vortices in order to reduce their flight energy when they fly in formation. This effect is expected to be applied to, e.g., flight of micro air vehicles. In the present study, a flow around two closely located airfoils is investigated by means of direct numerical simulation. As compared to a single flight case, the drag and the lift of the trailing airfoil are found to decrease and the lift-drag ratio of the leading airfoil is promoted in the vertical separation case. These in the separation case approach those in the single flight case. An interaction among the vortices induced by each airfoil is also observed. When the airfoils have a spanwise separation, the circulation of the airfoils does not remain constant and has varied in the streamwise direction. In the single airfoil case, its variation is caused dominantly by the viscous diffusion. The separation points on the airfoils move downstream, which contributes to the drag reduction. The lift coefficient distribution of the trailing airfoil becomes asymmetric in the spanwise direction due to modulation of the surface pressure distribution.

Key Words: Airfoil, Wing-tip vortex, Formation flight, Drag reduction

Nomenclature

\begin{tabular}{ll}
\textbf{Nomenclature} & \\
\(x, y, z\) & coordinate \\
\(u, v, w, U\) & velocity \\
\(P\) & pressure \\
\(t\) & time \\
\(C_d, C_l\) & drag and lift coefficients \\
\(c\) & chord length \\
\(AR\) & aspect ratio \\
\(Re\) & Reynolds number \(Re = U_*c / \nu\) \\
\(\nu\) & kinematic viscosity \\
\(Subscripts\) & \\
0 & single airfoil case \\
\(\infty\) & free stream \\
* & dimensional values \\
\(Superscripts\) & \\
\(\infty\) & uniform inlet velocity \\
\(\ast\) & \\
\end{tabular}

1. Introduction

Wing-tip vortices (WTVs) are generated during the flight of aircrafts. Because WTV cause the induced drag, extensive studies have been conducted to suppress them by a passive control such as a winglet\textsuperscript{1)} or an active control such as a plasma actuator or a synthetic jet\textsuperscript{2,3}. In the nature, however, certain birds save their flight energy by utilizing WTV via the V-shape formation\textsuperscript{4)}. It is expected that such an energy saving by WTV can be applied to the practical unmanned aerial vehicle (UAV) or micro air vehicle (MAV), which are used for investigation in a disaster. The formation flight enables UAV or MAV to overcome their shortcoming, viz. the limitation of the payload. However, the detailed effects of WTV on multiple airfoils at very low Reynolds number conditions are still unknown. In the previous study on the close flight, it is found that there is an important relationship between the airfoil performance and the location of the airfoils\textsuperscript{5).}

The objective of the present study is an investigation of the flow around two airfoils in formation flight and the relationship between their airfoil performance and the separation of the airfoils. In the present study, a three-dimensional direct numerical simulation (DNS) of the flow around two airfoils in formation flight is performed to investigate the effect of the vortices generated around the airfoils.

2. Direct numerical simulation

Governing equations are the continuity equation and Navier-Stokes equation, for an incompressible fluid. The Reynolds number is 3000, which is defined by the chord length \(c^*\) and the uniform inlet velocity \(U_\infty^*\):

\[
\nabla \cdot \mathbf{u} = 0
\]

\[
\frac{\partial \mathbf{u}}{\partial t} = -\nabla \cdot (\mathbf{u} \mathbf{u}) - \nabla p + \frac{1}{Re} \nabla^2 \mathbf{u}
\]

Fig. 1. (a) Computational domain; (b) Locations of two airfoils.
Table 1. Investigated cases and separation of two airfoils.

<table>
<thead>
<tr>
<th>Case No.</th>
<th>$h_z$</th>
<th>$h_x$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>2</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>0</td>
<td>-0.5</td>
</tr>
<tr>
<td>5</td>
<td>0</td>
<td>0.5</td>
</tr>
</tbody>
</table>

Table 2. Boundary conditions.

<table>
<thead>
<tr>
<th>Streamwise</th>
<th>Inlet</th>
<th>Uniform flow $U_\infty = 1.0$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Outlet</td>
<td>Velocity: convective outlet</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Pressure: NSCBC</td>
</tr>
<tr>
<td>Vertical</td>
<td>Free-slip</td>
<td></td>
</tr>
<tr>
<td>Spanwise</td>
<td>Periodic</td>
<td></td>
</tr>
<tr>
<td>Airfoil surface</td>
<td>Penalization method$^5$</td>
<td></td>
</tr>
</tbody>
</table>

The computational domain is $0 \leq x \leq 12, -3.5 \leq y \leq 3.5, -3 \leq z \leq 3$ shown in Fig. 1(a), and the grid number is $N_x \times N_y \times N_z = 256 \times 192 \times 128$. Figure 1(b) shows the locations of NACA0012 airfoils. Their aspect ratio is $AR = 2$, and the angle of attack is $10^\circ$. Whereas the trailing airfoils is fixed at $(x, y, z) = (4.5, 0, 0)$, the leading airfoil is fixed in the streamwise direction $(x)$ but is shifted in the vertical $(y)$ and the spanwise $(z)$ directions, respectively. The investigated cases in the present study are shown in Table 1.

The boundary conditions (B.C.) are shown in Table 2. The penalization method is used for the B.C. on the airfoil surface, so that the velocity inside the body or on the boundary becomes zero. The DNS code is extended from the external flow code of Hasebe et al.$^2$, which is based on the channel flow code of Fukagata et al.$^6$. As the spatial discretization, the TVD scheme$^7$ is used in streamwise direction and the energy-conservative second-order finite difference scheme is used in the other directions. The low storage third-order Runge-Kutta/Crank-Nicolson (RK3/CN) scheme is used as the time integration method. The Poisson equation for the pressure is solved by using the fast Fourier transform, the mirroring method, and the tridiagonal matrix algorithm (TDMA). The SMAC scheme is used for the coupling of velocity and pressure.

The drag and lift coefficients are defined as

$$
C_D = \frac{D^*}{\frac{1}{2}\rho U_\infty^2 S^*}, \quad C_L = \frac{L^*}{\frac{1}{2}\rho U_\infty^2 S^*},
$$

where $D^*, L^*, \rho^*$ represent the drag, the lift and the density of the fluid, respectively. The reference area $S^*$ is obtained as $S^* = AR c^{-2}$.

3. Results and discussion

3.1. Variation of drag and lift

Figure 2 shows the relationship between the variation of drag and lift and the location of the airfoils. The value is averaged by time and normalized by the value of the single airfoil case ($C_{D0}, C_{L0}$). Figure 2(a) shows that the drag of the leading airfoil decreases. Its lift-drag ratio, which represents the airfoil performance, is promoted at most 15% when the vertical separation between two airfoils has a large value. For the trailing airfoil, the drag decreases slightly but the lift also decreases; thus, the lift-drag ratio is unchanged. Figure 2(b) shows the variation of drag and lift when the airfoils are shifted in spanwise direction. There is little variation of the drag and the lift of the leading airfoil, and these value approaches those of the single airfoil case. Moreover, the drag and the lift of the trailing airfoil are recovered to those of the single airfoil case as the airfoils have a large separation in the spanwise direction.

Figure 3 shows the relationship between the separation of the airfoils and the fluctuations of the drag and the lift. When the leading airfoil is shifted in the vertical direction, the fluctuations of the leading airfoil is suppressed to about 40% as shown in Fig. 3(a). Regarding the drag and lift fluctuations of the trailing airfoil, they are suppressed as the leading airfoil shifts far from the trailing airfoil. However, these values are double those of the single airfoil case. When the leading airfoil shifts in the spanwise direction, there is small variation of the fluctuations of the leading airfoil shown in Fig. 3(b). The fluctuations of the trailing airfoil approach those of the single case when the spanwise separation between the airfoils has more than $h_z = 1$. 

![Image: Figure 2. Variation of mean drag and lift. (a) dependency on the vertical separation, $h_z$; (b) dependency on the spanwise separation, $h_x$.](image)

![Image: Figure 3. Variation of rms drag and lift. (a) dependency on the vertical separation, $h_z$; (b) dependency on the spanwise separation, $h_x$.](image)
3.2. Visualization of vortical structures and vortex strength

Figure 4 shows the vortical structure around the airfoils visualized by the second invariant of the velocity gradient tensor, $Q = -3.0$. In the single airfoil case shown in Fig. 4(a), it is found that the WTV remains downstream compared with the Kármán’s vortex structures. The vertical structures when the airfoils fly in formation at the location of Case 5 and Case 2 are shown in Fig. 4(b) and (c), respectively. The Kármán’s vortex structures collapse and the WTV are meandering.

To compare the strength of the WTV quantitatively, the circulation is computed:

$$\Gamma = \int_S \omega_x dS,$$

where $\omega_x$ represents the streamwise component of the vorticity.

Figure 5 shows the distribution of the circulation in the streamwise direction. The circulation in Case 1 ($h_i = 0$) is decreased than that in the single airfoil case. Therefore, it is considered the vortical strength is weakened by the formation flight. On the other hand, the circulation in Cases 2 and 3, in which the leading airfoil is shifted in spanwise direction, is not constant and depends on the streamwise position. The streamwise evolution of $\Gamma$ is investigated in detail by using the vorticity transport equation. It is obtained by the surface integral of the vorticity transport equation:

$$\frac{\partial \Gamma}{\partial t} = \int_S \omega_x \frac{\partial u}{\partial x} dS + \int_S \left( \omega_x \frac{\partial u}{\partial y} + \omega_y \frac{\partial u}{\partial z} \right) dS$$

$$+ \nu \int_S \nabla^2 \omega_x dS$$

The terms in the right hand side are denoted as the stretching term, the tilting term, and the viscous diffusion term, respectively.

Figure 6 shows the variation of each term in the single flight case. The stretching term increases behind the airfoil, but thereafter it decreases immediately to zero. Since the tilting term has varied only behind the airfoil, it is conjectured that the WTV has a circular cross section far from the airfoil. From the relationship between the area of WTV and the maximum vorticity of WTV in this case, shown in Fig. 7, it is found that the decrease of the area and the increase of the maximum vorticity make the diffusion term constant. As the diffusion term takes the largest value among those three terms in all of the cases, the variation of
the circulation is mainly caused by the viscous diffusion.

3.3. Pressure field and separation point

The drag is decomposed into the pressure drag and the friction drag. The pressure drag on the both airfoils in Case 5 is decreased from 78% to 65% compared with the single airfoil case. On the other hand, the friction drag on the leading airfoil increases only 5% and that on the trailing airfoil is unchanged. It is conjectured that the drag reduction in the Case 5 is caused by the decrease of the pressure drag.

Figure 8 shows the mean velocity and pressure field in the single airfoil case and Case 5. The pressure around the leading edge of the trailing airfoil decreases more than that of the single case. The separation points of the airfoils, denoted as cross symbols in Fig. 8, are shifted downstream. The pressure on the suction surface of the both airfoils in Case 5 has a smaller negative value than that in the single airfoil case. It is conjectured that these modifications cause the reduction in the pressure drag.

3.4. Lift distribution for spanwise direction

The lift is also decomposed into the pressure lift and the friction lift generated by the shear stress. The pressure lift is much larger than the friction lift. It represents the pressure lift dominates the variation of the lift. Figure 9 shows the lift coefficient distribution in the spanwise direction in Case 2 \((h_2 = 1)\). The spanwise distribution of the leading airfoil is symmetric and gives good agreement with that of the single airfoil case. However, the lift on the trailing airfoil has the large variation around the wing-root \((z = 0)\) where the leading WTV is overlapped. This phenomenon occurs by the variation of the pressure contribution.

Figures 10 and 11 show the spanwise pressure distribution on the airfoil surfaces in the single case and
Case 2, respectively. The pressure on the suction airfoil surface takes negative values in the single airfoil case, as shown in Fig. 10. The pressure at the leading edge is high due to a stagnation point, and it is gradually decreasing from the leading edge toward the trailing edge.

In the Case 2 shown in Fig. 11, it is found for the leading airfoil that the pressure near the leading edge on the suction surfaces increases (Fig. 11(a-1)). However, the pressure near the leading edge on the pressure surface, shown in Fig. 11(a-2), decreases compared with that in the single case. Therefore, the pressure difference between the suction and the pressure surfaces is almost unchanged and the lift of the leading airfoil takes a same value as that of the single case. For the trailing airfoil, the pressure near the leading edge on the suction surface edge increases in the region of \( z > 0 \), as shown in Fig. 11(b-1). It reduces the pressure difference between the suction and the pressure surfaces, so that the lift of the trailing airfoil decreases. On the other hand, Fig. 11(b-2) shows that the pressure near the leading edge on the pressure surface decreases. Especially, the decrease of the pressure in the region of \(-0.5 < z < 0\) has a large value, where a downwash flow is generated by the leading WTV. It is conjectured for the trailing airfoil that the pressure difference between the suction and the pressure surfaces is reduced in the downwash region, which brings the more lift reduction than that in the other region.

4. Conclusions

Direct numerical simulations of the flow around two airfoils in formation flight are performed in order to investigate the relationship between the location of airfoils and their airfoil performance. The lift-drage ratio of the leading airfoil is promoted when the airfoils have the vertical separation. The fluctuations of the drag and the lift, however, increase compared with those in the single airfoil case. When the leading airfoil shifts in the spanwise direction, the lift-drage ratio varies slightly. The fluctuations of the drag and the lift approach those in the single case as the spanwise separation becomes large. Moreover, the strength of WTV is dominated by the viscous diffusion. It is varied depending on the streamwise direction in the cases of spanwise separations. It is found that the drag reduction causes a shift of the flow separation point and the change of the pressure distribution on the airfoil surface. While the lift distribution of the leading airfoil is unchanged, that of the trailing airfoil decreases compared with that of the single case. The downwash velocity generated by the WTV affects the spanwise distribution of the pressure lift. The variation of the pressure on the airfoil surfaces causes the variation of the lift when the airfoils fly in formation.

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References