RESEARCH ARTICLE

Direct numerical simulation of spatially developing turbulent boundary layer for skin friction drag reduction by wall surface-heating or cooling

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Direct numerical simulation (DNS) of zero-pressure-gradient spatially developing turbulent boundary layer with uniform heating or cooling is performed aiming at skin friction drag reduction. The Reynolds number based on the free-stream velocity, \( U_1 \), the 99% boundary layer thickness at the inlet, \( \delta_0 \), and the kinematic viscosity, \( \nu \), is set to be 3000 and the Prandtl number is 0.71. The computational domain is set to be \( 9\pi\delta_0 \times 3\delta_0 \times \pi\delta_0 \) in the streamwise, wall-normal and spanwise directions, respectively. A constant temperature is imposed on the wall. The Richardson number \( Ri \) for the buoyancy is varied in the range of \(-0.1 \leq Ri \leq 0.1\). The DNS results show that uniform cooling (UC) reduces skin friction drag with a maximum drag reduction rate of 65%, while uniform heating (UH) enhances it. The trend is similar to that in channel flow studied by Iida and Kasagi (1997) and Iida et al. (2002) and that in spatially developing boundary layer flow by Hattori et al. (2007). Dynamical decomposition of skin friction drag using the identity equation (FIK identity, Fukagata et al. 2002) quantitatively shows that drag reduction by UC is due to reduced Reynolds shear stress (RSS), while drag increase by UH is augmentation of RSS. The control efficiency of UC, however, is found to be largely negative; namely, net power saving is not achieved.

Keywords: turbulent boundary layer; drag reduction; direct numerical simulation; control

1. Introduction

Reduction of skin friction drag in turbulent flow is a significant issue for mitigating the environmental impact of high speed transports such as aircraft, trains, and ships, through reduction of their fuel consumption. Despite the extensive research conducted, a practical method for skin-friction drag reduction is still being explored.

To date, various approaches for the skin friction drag reduction have been examined, triggered by the emergence of direct numerical simulation (DNS) of wall-bounded turbulent flow [1] in late 1980s: feedback control [2–5]; active predetermined control [6–8]; and passive control using structured roughness [9], compliant surface [10, 11], and additives [12]. However, most of previous numerical studies on friction drag reduction have dealt with internal flows such as channel and pipe flows. Toward the practical applications, control of external flows such as a spatially developing boundary layer needs to be investigated.

Pamiès et al. [13] performed large eddy simulations of spatially developing turbulent boundary layer using the opposition control of Choi et al. [14]. They also

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studied the case of uniform blowing (UB) in addition to the opposition control, with which a larger drag reduction than the opposition control alone was achieved. We have also performed DNS of spatially developing turbulent boundary layer with uniform blowing (UB) or suction (US) aiming at skin friction drag reduction [15]. It is found that UB reduces the drag and US increases it. Moreover, the mechanism of drag reduction by UB is found to be somewhat different from that by most of drag reduction control schemes: the drag reduction by UB in spatially turbulent boundary layer is achieved by a vertical mass flux from the wall but not by the suppression of turbulence. Although UB can simply be implemented in numerical simulations, it is difficult to be implemented in practice. Therefore, it is necessary to explore some other methods.

A possible option is a use of buoyancy. Iida and Kasagi [16] and Iida et al. [17] performed DNS of turbulent channel flow under stable and unstable density stratification, respectively. They found that under a weakly unstable density stratification skin friction drag was reduced due to suppression of streamwise vortices near the wall. They also showed that it is possible relaminarize the flow at a large amplitude of Richardson number. The direct numerical simulation of spatially developing turbulent thermal boundary layer under stable/unstable stratification was investigated by Hattori et al. [18] to assure the dissipation of contaminant in the atmospheric phenomena. The results were found to be in accordance with the finding in a channel flow. These results suggest the possibility of turbulence control using the buoyant force with uniform surface heating/cooling in external turbulent flows. This uniform heating or cooling is considered easier to be implemented in practice than the uniform blowing.

In the present study, DNS of zero-pressure-gradient spatially developing boundary layer is performed with uniform heating (UH) and cooling (UC). Although the simulation itself is similar to that performed by Hattori et al. [18], we discuss the results from more control point of view. In particular, the mechanism of drag reduction or augmentation is analyzed by using the dynamical decomposition of skin friction drag [19] (FIK identity) and compared with the previous work with uniform blowing/suction [15]. We also compare the control efficiency with other control schemes for friction drag reduction.

2. Numerical procedure

2.1. Direct numerical simulation

The governing equations are the incompressible continuity, Navier-Stokes and energy equations, i.e.,

\[ \frac{\partial u_i}{\partial x_i} = 0 \],

\[ \frac{\partial u_i}{\partial t} = -\frac{\partial u_i u_j}{\partial x_j} - \frac{\partial p}{\partial x_i} + \frac{1}{Re\delta_0} \frac{\partial^2 u_i}{\partial x_j \partial x_j} + Ri(1 - \theta), \]

\[ \frac{\partial \theta}{\partial t} = -\frac{\partial u_j \theta}{\partial x_j} + \frac{1}{Re\delta_0 Pr} \frac{\partial^2 \theta}{\partial x_j \partial x_j}, \]

where \( x_i \) and \( u_i \) (\( i = 1, 2, 3 \)) are the Cartesian coordinates and corresponding velocity components, respectively, and \( \theta \) is temperature. All variables are nondimensionalized by the freestream velocity \( U_\infty \), the 99% boundary layer thickness at the inlet of the computational domain \( \delta_0 \), and the temperature difference between the
Figure 1. Computational geometry

freestream and the wall $\Delta \theta$. The Reynolds number is defined as $Re_0 = U_{\infty} \delta_0 / \nu$, where $\nu$ is the kinematic viscosity. The Prandtl number is defined as $Pr = \kappa / \nu$, where $\kappa$ is the thermal conductivity. The Richardson number is defined as

$$Ri = \frac{g \beta \delta_0 \Delta \theta}{U'^2_{\infty}}, \quad (4)$$

where $g$ and $\beta$ denote the gravitational acceleration and the coefficient of volumetric expansion, respectively. The present DNS code is based on the spatially developing boundary layer flow code of Kametani and Fukagata [15]. The energy conservative second-order finite difference scheme (e.g. [20]) is adopted for the spatial discretization. The time integration uses the low-storage third-order Runge-Kutta/Crank-Nicolson scheme (e.g. [21]). The pressure Poisson equation is solved by using the fast Fourier transform with the mirroring technique in the streamwise direction [22].

In order to perform the simulation with control input (i.e., heated/cooled wall), the computational domain is decomposed into two parts: a driver region and a main region, as shown in Fig. 1. The recycle method of Lund et al. [23] is applied to the driver region to generate turbulent inflow condition. Unlike the DNS of Hattori et al. [18], who used the recycle method for the temperature also in the driver region, we assume isothermal flow in the driver region and the temperature is only solved in main region in order to study the effect of heating/cooling control. The inlet boundary condition of the temperature in main region is set to be constant ($\theta = 1$), assuming no temperature difference between the freestream and the uncontrolled wall.

In both driver and main regions, the upper boundary condition for streamwise velocity $u$, the wall-normal velocity $v$, and the spanwise velocity $w$ are set to be $\frac{\partial u}{\partial y} = \frac{\partial v}{\partial y} = 0$ and $w = 0$. On the wall, the no-slip condition is applied to each velocity component. As for the temperature in the main region, $\theta = 1$ at the upper boundary and $\theta = \theta_{ctr}$ on the wall. Since the temperature is nondimensionalized by the temperature difference between the freestream and the wall, $\theta_{ctr} = 0$ except for the transition region, as shown in Fig. 2. At the downstream end of each computational domain, the convective boundary condition is applied for each velocity component and temperature, i.e.

$$\frac{\partial f}{\partial t} + \bar{u}(y) \frac{\partial f}{\partial x} = 0 \text{ ,} \quad (5)$$

where $f$ is the velocity or temperature and $\bar{\cdot}$ denotes the average in the homogeneous
Figure 2. Profile of control input.

direction. For the pressure at the inlet and outlet boundary of each computational domain, the Navier-Stokes characteristic boundary condition (NSCBC) of Miyauchi et al. [24] is applied, i.e.,

\[
\frac{\partial p}{\partial t} + U_1 \frac{\partial p}{\partial x} = \frac{1}{Re_0} \omega_z^2,
\]

where \(\omega_z\) denotes the spanwise vorticity.

The streamwise, wall-normal and spanwise lengths of the driver and main regions are \((L^D_x, L^D_y, L^D_z) = (3\pi \delta_0, 3\delta_0, \pi \delta_0)\) and \((L_x, L_y, L_z) = (9\pi \delta_0, 3\delta_0, \pi \delta_0)\), respectively, where the superscript \(D\) denotes the driver region. The corresponding numbers of grid points are \((N^D_x, N^D_y, N^D_z) = (128, 96, 128)\) and \((N_x, N_y, N_z) = (512, 96, 128)\), respectively.

The recycle station is located at \(x^D = 2\pi \delta_0\). In this study, the Reynolds number is set to be \(Re = U_\infty \delta_0 / \nu = 3000\), where \(\delta_0\) denotes the 99% boundary layer thickness at the inlet of the main region, and the Prandtl number is assumed to be \(Pr = 0.71\). The Reynolds number based on the friction velocity at the inlet, \(u_\tau\), is \(Re_\tau \approx 160\). The minimum grid spacing in wall-normal direction is \(\Delta y^+ = 0.47\) and the maximum spacing is \(\Delta y^+ = 6.67\).

The statistics are accumulated over a time period of \(T^+ \approx 4000\) after a statistically steady state has been reached.

2.2. Wall surface-heating/cooling

In order to use the buoyant force as a control medium, uniform wall-heating (UH) or cooling (UC) is applied. The buoyancy is taken into account by the Boussinesq approximation. In the present study, the magnitude of UH and US is set to be \(Ri (Gr) = \pm 0.01 (0.9 \times 10^5), \pm 0.02 (1.8 \times 10^5), \pm 0.1 (9 \times 10^5)\), where positive sign denotes heating and negative sign denotes cooling. The corresponding Grashof number is given by \(Gr = Ri Re^2\). Hattori et al. [18] performed DNS with the thermal rescaling method of Kong et al. [25] to create the inlet profile of temperature.

In the present study, in contrast, we aim at using buoyancy as a control medium; therefore, uniform temperature is introduced at the inlet and a different wall-temperature is set in the main region. For smooth transition from the freestream temperature \((\theta = 1)\) to the wall temperature \((\theta = 0)\), a transition zone is located at \(0 \leq x \leq \pi\), in which the temperature is gradually varied using a hyperbolic tangent function, as shown in Fig. 2.
3. Results and discussion

3.1. Turbulence statistics

The effects of wall-heating/cooling on the spatial development of boundary layer thickness are shown in Fig. 3 as Reynolds numbers. Here, the momentum thickness $\delta_m$ and the enthalpy thickness $\delta_\Delta$ are defined as

$$\delta_m = \int_0^\infty U(1-U)dy$$

$$\delta_\Delta = \int_0^\infty U(1-\Theta)dy$$,

where $U$ and $\Theta$ denote the mean velocity and temperature, respectively. These profiles show that the momentum thickness is thickened by wall heating, while thinned by cooling. Similar trends appear in the development of thermal boundary layer; viz. the development of the thermal boundary layer is promoted by wall heating, while suppressed by cooling. The magnitude of both effects depends on the Richardson number. The iso-surface of instantaneous temperature is presented in Fig. 4. It is found that UH increases the thermal fluctuations by forming unstable stratification, while UC decreases it by forming stable stratification: the turbulence behaves as if its effective Reynolds number were increased (decreased) by the UH (UC) control. These trends are in accordance with the observation by Hattori et al. [18].

Figure 5(a) shows the local friction coefficient $c_f$ as a function of streamwise distance $x$ from the inlet, defined as

$$c_f = \frac{\tau_w}{\frac{1}{2} \rho U^2}$$,

where $\tau_w = \mu (\partial U / \partial y)_w$. It is found that the skin friction drag is reduced by UC, while enhanced by UH. The amplitude of buoyancy, $|Ri|$, affects on the profiles: the large amplitude results in larger reduction/enhancement of skin friction drag.
Figure 4. Iso-surfaces of temperature $\theta = 0.7$: (a) no control; (b) $Ri = 0.1$; (c) $Ri = -0.1$. 
Figure 5(b) presents the local Stanton number, defined as

\[ St = \frac{q_w}{\rho c_p U_\infty \Delta \theta}, \]  

where \( c_p \) denotes specific heat and \( q_w = \kappa (\partial \Theta / \partial y)_w \). Since the thermal boundary layer forms from the starting point of control, the Stanton number is quite large at the upstream location, i.e. \( Re_{\delta_{m,nc}} < 360 \) (where the subscript \( nc \) denotes the value in the uncontrolled case). In the downstream region, a similar trend to that for the local friction coefficient is noticed. These trends in \( c_f \) and \( St \) are basically similar to those previously reported for a channel flow \([16, 17]\) and a spatially developing boundary layer \([18]\) (although the Reynolds number assumed by Hattori et al. \([18]\) is higher than the present study).

Figures 5 (a) and (b) also show that friction coefficient reaches a curve of fully-heated/cooled turbulent boundary layer around \( Re_{\delta_{m,nc}} \geq 430 \) \((x \geq 13 \delta_0)\). Small oscillation observed near the downstream end of present computational domain, \( Re_{\delta_{m,nc}} \geq 530 \), especially in the UH cases is likely to be due to numerical instability. The Reynolds analogy factor \( 2St/c_f \) is shown in Fig. 5(c). Although the analogy factor is around unity in fully-developed flow, it is larger than unity in the present cases. This is because in the present simulations the onset of thermal boundary layer is more downstream than that of the velocity boundary layer, and the thermal boundary layer is always thinner than the velocity boundary layer. As compared to the uncontrolled case, the analogy factor is found to be slightly smaller in UH cases and larger in UC cases.

Figure 6 shows the drag reduction rate \( R \) calculated by using the global friction coefficient, as

\[ R = \frac{C_{f,nc} - C_{f,ctr}}{C_{f,nc}}, \]  

where

\[ C_f = \frac{1}{L_{ctr}} \int_0^{L_{ctr}} c_f(x) \, dx \]  

with the subscripts of \( nc \) and \( ctr \) denoting the uncontrolled and controlled cases, respectively, and \( L_{ctr} \) being the streamwise length of the controlled region. This indicates that larger amplitude of control achieves higher drag reduction (enhancement) by UC (UH). In the present study, \( R \approx 65\% \) is achieved in UC case at \( Ri = -0.1 \), while \( R \approx -30\% \) in UH at \( Ri = 0.1 \). In the range between \(-0.02 \leq Ri \leq 0.02\), the figure suggests that there is a nearly linear relationship between the control amplitude and the drag reduction rate.

The mean streamwise velocity profiles at the location of \( Re_{\delta_{m,nc}} = 430 \) are shown in Fig. 7. The mean velocity is also nondimensionalized by the wall units of the uncontrolled case. As compared to the uncontrolled case, the profiles are shifted away from the wall by UC and toward the wall by UH.

The root-mean-square (rms) of each velocity component at \( Re_{\delta_{m,nc}} = 430 \) is shown in Figs. 8(a)-(c). Obviously, the turbulence is suppressed by UC and enhanced by UH. The streamwise velocity fluctuations are more significantly influenced by the strong cooling. The peaks shift to the wall by UH, while away from the wall by UC. A second peak appears at \( 60 \leq y^{+nc} \leq 110 \) for UH at \( Ri = 0.1 \). This second peak becomes clearer as the Reynolds number is increased (not shown). The wall-normal fluctuations are directly influenced by the buoyancy for its direction.
Figure 5. Control effects on (a) friction coefficient, (b) Stanton number, and (c) analogy factor, $2St/c_f$.

Black, no control; red, $Ri = 0.1$; magenta, $Ri = 0.02$; yellow, $Ri = 0.01$; green, $Ri = -0.01$; light blue, $Ri = -0.02$; blue, $Ri = -0.1$.

Figure 6. Drag reduction rate as a function of Richardson number at $Re_{\delta_{m,nC}} = 430$. 
Therefore, UH and UC with the same magnitude augment and suppress it almost equally. The peaks remain in the log-law region ($40 \leq y^{+} \leq 100$). The trend for the spanwise fluctuations is found to be similar to that for the wall-normal fluctuations. In addition, the spanwise fluctuations are observed to be influenced by UH/UC in the region closer to the wall than that for the wall-normal fluctuations.

The Reynolds shear stress and viscous shear stress are shown in Fig. 8(d). It can be seen that UC reduces the viscous shear stress, while UH enhances it. The Reynolds shear stress is also reduced by UC, while increased by UH. In UC at $Ri = -0.1$, the flow is almost relaminarized and the viscous shear stress is dominant. These results suggest that the vortical motion in vicinity of the wall is suppressed and the flow is stabilized by UC, while UH destabilizes the flow. Namely, as is well known, UC forms stable density stratification, while UH does unstable one.

The mean and rms temperatures are shown in Figs. 9(a) and (b), respectively. Since the thermal boundary layer begins to form in the upstream region of the computational domain, its thickness is thin compared to the momentum thickness; therefore, log-law region is not clearly observed in the mean temperature profile. Apart from that difference, similar trends to those for the streamwise mean velocity are observed: the profiles are shifted toward the wall by UH and away from the wall by UC. Similarly, the thermal fluctuations are promoted by UH, while suppressed by UC. Figure 9(c) shows the turbulent heat flux: the streamwise, $-u'\theta'$, and the wall-normal, $-v'\theta'$, components. The intensities are affected by UH/UC similarly to those of the streamwise velocity fluctuations. In the log-law region of mean velocity, however, the temperature fluctuations rapidly vanish. This is, again, because in the present simulations the thermal boundary layer is always thinner than the velocity boundary layer. The streamwise turbulent flux takes negative value, while the wall-normal flux takes positive value. With UC (UH), their magnitudes are decreased (increased). These results also support the argument that turbulence is suppressed (enhanced) by UC (UH). The peaks of the streamwise turbulent heat flux shift toward wall by UH and away from the wall by UC in the buffer layer, while those of wall-normal flux almost remain in the log-law layer. In the budget of the Reynolds shear stress, an additional term, $-Ri \bar{w}\theta'$, appears. In UH cases, i.e. positive Richardson numbers, this term works as a gain for the Reynolds shear stress (as mentioned in Hattori et al. [18], too), while the opposite in UC cases.
These modifications of streamwise turbulent heat flux and the shift of its peak indirectly affects the change in skin friction drag via the change of the Reynolds shear stress.

Figure 10 shows the budgets of turbulent kinetic energy:

\[ 0 = C_k + P_k + D_k^p + D_k^\mu + D_k^T + \epsilon_k + B_k, \]  

where terms in right hand side denote the convection term, the production term, pressure diffusion term, the viscous diffusion term, the turbulent diffusion, the dissipation and the buoyancy term, in order. For the computation of budgets, the consistent scheme by Mamori and Fukagata [26] is used. In the uncontrolled case, the convection term is quite small, which indicates that the contribution from each term is similar to that in a channel flow. The buoyancy term appears as a gain factor in UH case \((Ri = 0.1)\), and a loss factor in UC case \((Ri = -0.1)\). Accordingly, the turbulent kinetic energy is increased in UH case, while decreased in UC case. In the UC case, all terms are much smaller than those of uncontrolled flow, which leads toward relaminarization especially near the wall \((y^{+nc} \leq 10)\).
3.2. **Statistical decomposition of \( c_f \) by FIK identity**

Dynamical decomposition of skin friction coefficient is performed by using the FIK identity \[19\] in order to clarify the contributions of different effects on the change of skin friction. For the spatially developing plane turbulent boundary layer, the local skin friction coefficient \( c_f \) is decomposed into five different dynamical contributions: the contributions from boundary layer thickness, \( c_\delta \), the Reynolds shear stress, \( c_T \), mean convection, \( c_C \), spatial development, \( c_D \), and pressure gradient (due to buoyancy, as explained below), \( c_P \), i.e.,

\[
\begin{aligned}
c_f(x) &= \frac{4(1 - \delta_d)}{Re_\delta} + 2 \int_0^1 2(1 - y) (-u'v') \, dy + 2 \int_0^1 2(1 - y) (-UV) \, dy \\
&- 2 \int_0^1 (1 - y)^2 \left( \frac{\partial U}{\partial x} + \frac{\partial u'v'}{\partial x} - \frac{1}{Re} \frac{\partial^2 U}{\partial x \partial x} \right) \, dy - 2 \int_0^1 (1 - y)^2 \left( \frac{\partial P}{\partial x} \right) \, dy, \quad (14)
\end{aligned}
\]
where $\delta_d$ is the dimensionless displacement thickness and all the variables are nondimensionalized by the freestream velocity $U_\infty$ and the local 99% boundary layer thickness $\delta$. By integrating local friction coefficient in the streamwise direction, the global friction coefficient $C_f$ is also decomposed as

$$C_f = \frac{1}{L_{ctr}} \int_0^{L_{ctr}} c_f(x) dx = \frac{1}{L_{ctr}} \int_0^{1} \left( c^\delta(x) + c^T(x) + c^C(x) + c^D(x) + c^P(x) \right) dx = C^\delta + C^T + C^C + C^D + C^P.$$  \hspace{1cm} (15)

These equations indicate two main directions to reduce the skin friction drag: suppression of the Reynolds stress term and enhancement of the mean convection term. An example of the former is a turbulence control aiming at suppression of quasi-streamwise vortices, such as the opposition control [14, 19]. On the other hand, an example of the latter is a spatially developing turbulent boundary layer with uniform blowing from the wall [15].

Figure 11(a)-(c) presents the decomposed local skin friction coefficient in the uncontrolled and controlled cases at $Ri = \pm 0.02$. All cases have a similar balance, except for the pressure gradient term: $c^\delta$, $c^T$, and $c^D$ are the enhancement factors.
Figure 11. Each term of the FIK identity: (a) no control, (b) $Ri = 0.1$, (c) $Ri = -0.1$. Black, $c_f$ calculated from the mean streamwise velocity gradient on the wall; red, $c^\delta$; blue, $c^T$; green, $c^C$; magenta, $c^D$; gray, $c^P$ ($= c_f - c^\delta - c^T - c^C - c^D$).

for the skin friction drag, while $c^C$ is the reduction factor. In the uncontrolled case, the pressure gradient term $c^P$ is zero. The small deviation near the inlet and outlet is due to the boundary condition. However, non-zero $c^P$ is generated by UC and UH. In UC case, the cooled bulk fluid is accelerated downward due to gravity. This nearly homogeneous downward acceleration should mostly be canceled by the wall-normal pressure gradient. Thus, the pressure near the wall increases to generate negative $c^P$. The positive $c^P$ in UH case can also be explained likewise. Note that, unlike the cases of uniform blowing or suction, the mean wall-normal velocity should hardly be affected directly by UC or UH due to the impermeable condition on the wall and the incompressibility constraint.

The contributions to the global friction coefficient for the uncontrolled, uniform heating (UH) and cooling (UC) cases at $Ri = \pm 0.02$ are compared in Fig. 12. It is clearly shown that UC reduces friction drag by reducing the Reynolds stress term $C^T$ and enhancing the mean convection term $C^C$. It is also clear that $C^C$ has a negative contribution; namely, it works as a drag reduction factor. The pressure gradient term is smallest in each case, but it grows as the control amplitude becomes higher. The summation of the mean convection term $C^C$ and the spatial development term $C^D$ (which is originally defined as the spatially development...
Figure 12. Decomposed global friction coefficient by FIK identity ($10^{-3}$).

term in Fukagata et al. [19]), are almost equal in both UH and UC cases. Therefore, the effect of drag reduction or enhancement mostly comes from the change in the Reynolds stress term $C^T$. This is clearly different from the cases of uniform blowing (suction) [15], where the major contributor to the friction drag reduction (enhancement) is the negative (positive) mean convection term.

### 3.3. Control efficiency

In practical applications, it is necessary to consider the efficiency of control. Considering only an ideal control input (viz., neglecting any mechanical loss in actuators/sensors), the drag reduction rate $R$, gain $G$, and net energy saving rate $S$ can be defined as (see, e.g. [27])

$$R = \frac{W_0 - W}{W_0},$$
$$G = \frac{W_0 - W}{W_{\text{in}}},$$
$$S = \frac{W_0 - (W + W_{\text{in}})}{W_0},$$

where $W_0$ and $W$ are the driving power in the uncontrolled and controlled cases, respectively, and $W_{\text{in}}$ denotes the power of the ideal control input. Figure 13 illustrates a schematic of the relationship between $R$ and $S$. These measures are evaluated by using the global skin friction coefficients, as

$$G = \frac{C_{f,\text{nc}} - C_{f,\text{ctr}}}{W_{\text{in}}/L_{\text{ctr}}},$$
$$S = \frac{C_{f,\text{nc}} - (C_{f,\text{ctr}} + W_{\text{in}}/L_{\text{ctr}})}{C_{f,\text{nc}}}. $$

In the present cases, the driving powers for the flows with control $W$ and without control $W_0$ are equivalent to $C_{f,\text{ctr}}L_{\text{ctr}}$ and $C_{f,\text{nc}}L_{\text{ctr}}$, respectively. The input power,
Figure 13. Schematic of control efficiency

$W_{in}$, for the uniform heating/cooling is computed as

$$W_{in} = \frac{1}{Re \, Pr} \int_0^{L_{ctr}} \left( \frac{\partial \theta}{\partial y} \right)_{w} \, dx.$$  \hfill (21)

Figure 14 shows the relationship between $G$ and $S$ computed using the DNS data. The values reported in the previous studies for channel flows and uniform blowing (UB) control in spatially developing boundary layer [15] are also shown for comparison. It is clearly seen that the net energy saving rate is largely negative in all uniform cooling (UC) cases examined in the present study; namely, the control requires more power than it can save the driving power.

In order to clarify the reason for this low efficiency, the kinetic energy generated by the bouyancy, $W_{in}'$, is calculated as [28]

$$W_{in}' = \int_{\mathcal{V}} |Ri| \, \bar{v}(1 - \bar{\theta}) \, d\mathcal{V},$$  \hfill (22)

where $\mathcal{V}$ denotes the computational domain. Figure 15 shows the conversion ratio $W_{in}' / W_{in}$ at different Richardson numbers. The conversion ratio is found to be very small and mildly increases with the Richardson number. Even at $Ri = 0.1$, only 13.5% of thermal energy is converted into kinetic energy and the rest is simply convected away.

4. Conclusions

We performed DNS of zero-pressure-gradient turbulent plane boundary layer flow at $Re_{\tau,0} \approx 160$ with uniform cooling/heating aiming at reduction of skin friction drag. In this low Reynolds number flow, the uniform cooling achieved 65% friction drag reduction, while heating resulted in 30% drag increment.

From the shear stress profiles, it is found that uniform cooling reduces both of the viscous shear stress and the Reynolds shear stress, while uniform heating has the opposite trend.

The mechanism of skin friction drag reduction by uniform cooling is found to be different from that by uniform blowing in previous study of Kametani and Fukagata [15]. The cooling control achieves drag reduction by reducing the vortices near the wall, i.e. reducing the Reynolds stress term in the FIK identity, while the uniform blowing does it by blowing the vortices away from the wall, i.e. enhancing the mean convection term.
Figure 14. Net energy saving rate achieved by different active control schemes: *, Kametani and Fukagata’s uniform blowing at a different blowing amplitude [15]; o, Choi et al.’s opposition control [14] (computed by Iwamoto et al. at different Reynolds numbers [29]); +, Lee et al.’s suboptimal control [30] (Iwamoto et al. [29]); ×, temporally-periodic spanwise wall-oscillation (Quadrio and Ricco [31]); ⊙, streamwise traveling wave (Min et al. [7]); □, steady streamwise forcing (Xu et al. [32]); Δ, spatially-periodic spanwise oscillation (Yakeno et al. [33]). Solid circle markers denote UC in the present simulation: green, $Ri = -0.01$; light blue, $Ri = -0.02$; blue, $0.1\%$ $Ri = -0.1$.

Although skin friction drag is reduced by uniform cooling, the net energy saving rate is found to be largely negative; namely, net energy saving is not achieved. This is because only a small portion of thermal input is used to generate the buoyant force and the rest is convected away unused. The situation is considered more severe in practical high Reynolds number flows because an extremely large temperature difference will be required according to the definition of Richardson number ($Ri = Gr/Re^2$).

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